

$N = 2$ SUPERGRAVITY AND SPECIAL GEOMETRY¹

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ABSTRACT

The essential elements in the construction of the couplings of vector multiplets to supergravity using the conformal approach are repeated. This approach leads automatically to the basic quantities on which the symplectic transformations, the basic tools for duality transformations, are defined. A recent theorem about the existence of a basis allowing for a prepotential is discussed.

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1 Introduction

The construction of $N = 2$ supergravity and its matter couplings using superconformal tensor calculus was done in 1979–1984 [1, 2, 3, 4]. Later, other constructions [5] drew the attention to the freedom of choice of coordinates. The choice which we made in the first construction, called special coordinates, was not inherent to the setup, but rather it was the most natural one to choose. However, this choice has led to some confusion by the fact that coordinates and what are now called the symplectic vectors, were not clearly distinguished. The symplectic vectors occur most naturally in the conformal construction. It is therefore no surprise that symplectic transformations in $N = 2$ were first found during the investigation of the general couplings of vector multiplet couplings [3], nearly immediately after the construction of special Kähler geometry itself [2]. The latter is by definition the geometry determined by the kinetic terms of the scalars in the $N = 2$, $d = 4$ vector multiplets coupled to supergravity. Let me also mention that this approach is now also used to construct the couplings of supergravity to the vector-tensor multiplet, a coupling which occurs in the context of string theories [6].

The superconformal approach has also its shortcomings. One can not prove that the obtained action is the most general one. So far, no procedure has been found to construct the most general couplings of hypermultiplets. Probably some construction similar to the harmonic superspace [7] approach is required. Furthermore it has been found that formulations for vector multiplet couplings exist which have not been obtained from superspace or conformal tensor calculus [8]. In fact, the construction of the action in these approaches starts from a holomorphic prepotential. In [8], we constructed vector multiplet couplings for which there exists no prepotential. On the other hand, it has been found [9, 10], in the context of defining special Kähler manifolds in an intrinsic way, that any such manifold can be described by a prepotential function. The scalar manifold of any model for which no prepotential exists is also the scalar manifold of models *with* a prepotential, the relation being made by a duality transformation. However, it is possible that they are physically different because they allow different groups to be gauged or different couplings to hypermultiplets.

With all this, I thought that a review putting the above differences more explicit would be welcome. At the workshop, for pedagogical reasons I covered for a large part the rigid special geometry. This part of the talk can be found in [9], while a more extensive review of aspects of the vector multiplet couplings in rigid supersymmetry has been given in [11], whose first two appendices contain our conventions.

2 Superconformal tensor calculus

In this section, I will review the construction and the ingredients entering in the couplings of vector multiplets to supergravity. Let it be clear that our aim is not to construct actions invariant under the superconformal group, but the

class of actions (at most quadratic in derivatives of fields) which are invariant under the subgroup of the latter: the $N = 2$ super-Poincaré group, possibly extended by Yang-Mills gauge invariances.

The principle of superconformal tensor calculus is that one starts with defining multiplets of the larger group, and then at the very end breaks it down to the super-Poincaré group. This, at the first sight cumbersome, procedure in fact simplifies life. The Poincaré supersymmetric action contains many terms whose origin is not clear, unless one recognises how they combine in the superconformal setup in natural objects (e.g. superconformal covariant derivatives or auxiliary fields). Also, as we will see, the symplectic vectors, crucial for the understanding of duality symmetries, have a natural origin in the superconformal setup.

2.1 Example without supersymmetry

The conformal group contains translations P^a , Lorentz rotations M^{ab} , Weyl dilatations D and special conformal transformations K^a . These are realised on a 'Weyl multiplet', consisting of the independent fields e_μ^a , called the vierbein, and b_μ , the gauge field of dilatations. The gauge fields of the Lorentz rotations and special conformal transformations are composite fields, resp.

$$\begin{aligned}\omega_\mu^{ab} &= -2e^{\nu[a}\partial_{[\mu}e_{\nu]}^{b]} + e^{a\rho}e^{b\sigma}e_{\mu c}\partial_{[\rho}e_{\sigma]}^c + 2e_\mu^{[a}b^{b]} \\ f_{\mu a} &= \frac{1}{4}(-R_{\mu a} + \frac{1}{6}e_{\mu a}R)\end{aligned}\tag{1}$$

with

$$R_{\mu\nu}{}^{ab} = 2\partial_{[\mu}\omega_{\nu]}^{ab} + 2\omega_{[\mu}{}^a{}_c\omega_{\nu]}^{cb}; \quad R_{\mu a} = R_{\mu\nu ba}e^{\nu b}; \quad R = R_{\mu a}e^{\mu a}.\tag{2}$$

The envisaged breakdown from superconformal to super-Poincaré symmetry will have to be done by 'compensator fields', i.e. fields whose value will be fixed in the course of this symmetry breaking. We illustrate this first with the simplest example. The simplicity has the disadvantage that one does not see the usefulness of the procedure, but the reader who wants to be convinced of that, should compare equations in super-Poincaré and superconformal approaches.

Consider one scalar field coupled in a conformal invariant way. The scalar field is taken to have Weyl weight 1, which means that it transforms under the Weyl dilatations with parameter Λ_D as $\delta\phi = 1.\phi\Lambda_D$. One then shows that the conformal invariant action is (see e.g. the exercises of lecture 2 in [12])

$$\mathcal{L} = \frac{1}{2}e\phi D_a D^a \phi = e\left(-\frac{1}{2}\partial_\mu\phi \cdot \partial^\mu\phi + \frac{1}{12}R\phi^2\right),\tag{3}$$

where a covariant derivative contains connections for M , D and K -transformations. The latter leads, via the expression Eq. 1, to the R -term. The dilatations are gauge fixed by fixing the value of ϕ . Taking it e.g. equal to $\sqrt{6}$, we obtain the Einstein gravity action with the canonical normalisation. b_μ has disappeared in Eq. 3. This is due to the K special conformal symmetry under which $\delta b_\mu = e_{\mu a}\Lambda_K^a$, while ϕ and e_μ^a are invariant. In other words b_μ can be fixed to zero as K -gauge choice.

2.2 $N = 2$ conformal tensor calculus

For $d = 4$, $N = 2$ the superconformal group is

$$SU(2, 2|N = 2) \supset SU(2, 2) \otimes U(1) \otimes SU(2) . \quad (4)$$

The bosonic subgroup, which I exhibited, contains, apart from the conformal group $SU(2, 2) = SO(4, 2)$, also $U(1)$ and $SU(2)$ factors. The Kählerian nature of vector multiplet couplings and the quaternionic nature of hypermultiplet couplings is directly related to the presence of these two groups, whose gauge fields will become the Kähler and quaternionic connections. On the fermionic side there are the two supersymmetries Q^i and two special supersymmetries S^i , which we will also have to gauge fix. For the gauge fixing of the superfluous gauge symmetries one makes use of a vector multiplet and a second compensating multiplet. For the latter three convenient choices exist. We will use here a hypermultiplet. The Weyl multiplet has as independent fields

$$\{e_\mu^a, b_\mu, \psi_\mu^i, A_\mu, \mathcal{V}_\mu{}^i{}_j, T_{ab}^-, \chi^i, D\} \quad (5)$$

These are the gauge fields for general coordinate transformations, dilatations, Q^i , $U(1)$, $SU(2)$ and, as extra fields to close the algebra, an antiselfdual anti-symmetric tensor, a doublet of fermions and a real scalar. The gauge field of S -supersymmetry is a composite, similar to those in Eq. 1, which by the way get extra terms now [1, 4]. As above, b_μ compensates for the K symmetry. The vierbein and the gravitinos deliver already all the physical fields one expects for pure $N = 2$ gravity except for the graviphoton, which sits in the vector compensating multiplet (see below).

A hypermultiplet contains 4 real scalar components and a doublet of spinors. After choosing the canonical action for this multiplet, the compensating hypermultiplet completely disappears as physical multiplet by the gauge choices of $SU(2)$ and the field equation of D , while the field equation of χ^i eliminates the fermions.¹

For a theory with n physical vector multiplets we thus introduce $n + 1$ vector multiplets to start with. These consists of fields (with (w, c) weights defining their transformation under the dilatation and $U(1)$ transformations in the superconformal group: $\delta\phi = w\phi\Lambda_D + i c\phi\Lambda_{U(1)}$)

$$\left(\begin{array}{cccc} X^I & \Omega_i^I & \mathcal{A}_\mu^I & Y_{ij}^I \end{array} \right)_{(1, -1), (\frac{3}{2}, -\frac{1}{2}), (0, 0), (2, 0)} \quad \text{with} \quad I = 0, 1, \dots, n. \quad (6)$$

The action is produced by the chiral superspace integral of a holomorphic function $F(X)$ as in rigid supersymmetry (see e.g. the review [11]). However, for

¹Although the hypermultiplet disappears in this way, it can play a non-trivial role. "Gauged $N = 2$ " and Fayet-Iliopoulos terms occur when the fields of the hypermultiplets are taken to transform under a gauge group (with vector multiplets to deliver the gauge fields). Then the gauge fixing of the scalars of the compensating hypermultiplet preserves a diagonal subgroup of the superconformal $SU(2)$ with the gauge group. As a result also e.g. the gravitino transforms under the unbroken gauge group, and a Fayet-Iliopoulos term occurs [3].

invariance under dilatations, this function F must now be homogeneous of second degree in X . (E.g. $F = (X^1)^3/X^0$ or $F = \sqrt{X^0}(X^1)^3$ is allowed).

The Y -fields (real $SU(2)$ triplets) will be auxiliary in the action. After gauge choices of dilatations, the $U(1)$ and S -supersymmetry, we will only be left with n complex scalars, n doublets of spinors, and $n+1$ physical vectors, including the gravitino as mentioned above. The reader can check that all superfluous symmetries are then broken by gauge choices.

The vectors \mathcal{A}_μ^I are gauge fields of an extra $n+1$ dimensional gauge group, for which all the fields of Eq. 6 are in the adjoint representation, and hypermultiplets may be in other representations. One of these possibilities was already alluded to in footnote 1. For the action to be invariant, the function F is not necessary invariant, because a real quadratic piece in F does not contribute to the action. Therefore we should have for the gauge symmetries with parameters y^I : [4]

$$\delta F \equiv g F_J f_{IK}^J X^K y^I = g y^I C_{I,JK} X^J X^K, \quad (7)$$

(for now F_I, F_{IJ}, \dots are the derivatives of F) where the C coefficients are real numbers².

Before taking a gauge choice, typical terms like those in Eq. 3 occur, now with R multiplied by (after elimination of the D auxiliary field, see [4]) $i(\bar{X}^I F_I - \bar{F}_I X^I)$. A standard Einstein action can be obtained by choosing as dilatational gauge fixing

$$i(\bar{X}^I F_I - \bar{F}_I X^I) = -X^I N_{IJ} \bar{X}^J = 1; \quad N_{IJ} \equiv 2 \operatorname{Im} F_{IJ} \quad (8)$$

At this point one could also use the 'string frame' by putting this equal to an exponential of a dilaton field if the latter is already well defined. This illustrates the flexibility of the superconformal approach.

3 Special Kähler geometry

3.1 Action, variables and Kähler-Hodge manifolds

Because of the constraint 8, the physical scalar fields parametrize an n -dimensional complex hypersurface. It is convenient to write first

$$X^I = a Z^I. \quad (9)$$

This introduces an extra variable and an extra gauge transformation, because we can redefine

$$a' = a e^{\Lambda_K(Z)}; \quad Z'^I = Z^I e^{-\Lambda_K(Z)}. \quad (10)$$

For reasons which will become clear soon, we will call this the 'Kähler transformation'. Because of the presence of this transformation, we can choose to have

²For transformations with non-zero C , the Kähler potential is not invariant [13], and this possibility has been omitted in [14]

Z invariant under the dilatation and $U(1)$ transformation, which then act on a . The constraint implies

$$|a|^2 = e^K(Z, \bar{Z}) \equiv i(\bar{Z}^I F_I(Z) - \bar{F}_I(Z) Z^I) = -\bar{Z}^I N_{IJ} Z^J, \quad (11)$$

(due to the homogeneity $F_I(X) = a F_I(Z)$, while $N_{IJ}(X) = N_{IJ}(Z)$), and the transformation Eq. 10 leads to a possible redefinition

$$K'(Z, \bar{Z}) = K(Z, \bar{Z}) + \Lambda_K(Z) + \bar{\Lambda}_K(\bar{Z}). \quad (12)$$

The action for the scalars is

$$\mathcal{L}_0 = -e N_{IJ} \mathcal{D}_\mu X^I \mathcal{D}^\mu \bar{X}^J \quad \text{with} \quad \mathcal{D}_\mu X^I = (\partial_\mu + i A_\mu) X^I. \quad (13)$$

After elimination of the auxiliary gauge field of the $U(1)$ transformation in the superconformal group,

$$A_\mu = \frac{i}{2} N_{IJ} (X^I \partial_\mu \bar{X}^J - \partial_\mu X^I \bar{X}^J), \quad (14)$$

and writing \mathcal{L}_0 in terms of Z^I , the phase of a disappears because of the $U(1)$. In other words we can take the gauge choice $a = \bar{a}$, which leaves a combination of the $U(1)$ and the Kähler transformation Eq. 10:

$$2i\Lambda_{U(1)} = \Lambda_K(Z) - \bar{\Lambda}_K(\bar{Z}). \quad (15)$$

The action then becomes of the Kählerian type

$$\mathcal{L}_0 = -\partial_\mu Z^I \partial_\mu \bar{Z}^J \frac{\partial}{\partial Z^I} \frac{\partial}{\partial \bar{Z}^J} K(Z, \bar{Z}). \quad (16)$$

The remaining Kähler transformation can e.g. be used to choose one of the Z^I , say Z^0 , equal to 1. In any case, one can choose the parametrization of the n physical scalars z^α (with $\alpha = 1, \dots, n$) at random, as stressed in [5]. To preserve the complex structure, already apparent in Eq. 16, the relations $Z^I(z^\alpha)$ should be holomorphic functions. E.g. a convenient choice for the inhomogeneous coordinates z^α at $Z^0 \neq 0$ are the *special* coordinates, defined by $Z^0 = 1$, $Z^\alpha = z^\alpha$. These were always used in the articles before [5]. The resulting geometry is known as *special* Kähler geometry [2, 3, 15]. There is one global aspect which is important, and as far as I know it is the only instant where the fermion sector comes in. As implied in Eq. 6, the fermions transform under the superconformal $U(1)$ factor, and hence, by Eq. 15, under the (finite) Kähler transformations:

$$\Omega_i \rightarrow e^{-\frac{1}{4}(\Lambda_K(Z) - \bar{\Lambda}_K(\bar{Z}))} \Omega_i. \quad (17)$$

Then one argues in the same way as for the magnetic monopole (as nicely explained in [16]): the fermion should remain well defined when going e.g. around a sphere. Therefore in going around in different patches, the reparametrisations should be such that $\Lambda_K(Z) - \bar{\Lambda}_K(\bar{Z})$ is $2\pi i$ times an integer. The properly

normalised gauge field is then $A_\mu/(4\pi)$ (using Eq. 15), and with Eq. 14 the field strength is

$$\Omega_{\mu\nu} = -\frac{i}{4\pi} \frac{\partial^2 K}{\partial Z^I \partial \bar{Z}^J} \partial_{[\mu} Z^I \partial_{\nu]} \bar{Z}^J, \quad (18)$$

hence, proportional to the Kähler 2-form. Kähler manifolds on which the transitions between coordinates in different patches can be done with such transformations with integer parameters are called 'Kähler-Hodge'.

Let us still note that the conditions for positive kinetic energies are that $g_{I\bar{J}} = \frac{\partial}{\partial Z^I} \frac{\partial}{\partial \bar{Z}^J} K$ is positive (with one zero mode), and that $Z^I N_{IJ} \bar{Z}^J$ is negative. That determines the positivity domain of the variables.

3.2 Duality transformations, intrinsic definition and existence of the prepotential

So far, we started from a particular function $F(X)$, but duality symmetries imply that different functions can lead to the same manifold. To understand the duality transformations, one has to consider the coupling to the vectors. This is governed by the complex symmetric tensor

$$\mathcal{N}_{IJ} = \bar{F}_{IJ} + i \frac{(N_{IN})(N_{JK})Z^N \bar{Z}^K}{(N_{LM})Z^L \bar{Z}^M}. \quad (19)$$

describing the coupling of the vector field strengths³. After some analysis of the fields equations, it turns out that duality transformations, leading to equivalent field equations (for abelian gauge fields), can be obtained by transformations in the group $Sp(2(n+1), \mathbb{R})$, and taking the quantisation conditions of charges into account, these transformations are restricted to integers. Under such transformations with a matrix

$$\mathcal{S} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \in Sp(2(n+1), \mathbb{R}), \quad (20)$$

\mathcal{N} should change to $(C + D\mathcal{N})(A + B\mathcal{N})^{-1}$. This can be obtained by considering

$$v = \begin{pmatrix} Z^I \\ F_I(Z) \end{pmatrix} \quad (21)$$

as a symplectic vector: $v' = \mathcal{S}v$. The Kähler potential Eq. 11 is obviously an invariant. The lower components of v' are again the derivatives of a scalar function $F'(Z')$ if the relation $Z'^I(Z) = A^I_J Z^J + B^{IJ} F_J(Z)$ is invertible, or in other words $A^I_J + B^{IK} F_{KJ}$ is invertible. $A + B\mathcal{N}$ is invertible in the positivity domain ($\text{Im } \mathcal{N} < 0$). In rigid supersymmetry $\mathcal{N}_{IJ} = \bar{F}_{IJ}$, and F' thus always exists. However, in the supergravity case important exceptions are known [8].

This implies that there are formulations where the prepotential F does not exist. These have thus been constructed by starting with a prepotential F ,

³Its imaginary part is negative definite in the positivity domain [17].

and then making a symplectic transformation. This is one of the arguments to formulate a definition of a special Kähler manifold without making reference to such a prepotential. That was already done by Strominger [15], but it turned out that his definition was not restrictive enough for allowing $N = 2$ supergravity [9, 10], which should be the decisive criterion to decide whether a manifold is special Kähler. A new definition has been given, and it was proven that all manifolds allowed by this new definition can be obtained starting from the above construction with a prepotential [9, 10].

One definition of a special Kähler manifold is as follows. It is an n -dimensional Hodge-Kähler manifold \mathcal{M} with a positive definite metric and with \mathcal{L} a complex line bundle whose first Chern class equals the Kähler form \mathcal{K} . There is a $Sp(2(n+1), \mathbb{R})$ vector bundle \mathcal{H} over \mathcal{M} , and a holomorphic section $v(z)$ of $\mathcal{L} \otimes \mathcal{H}$, such that the Kähler potential is given by Eq. 11, and such that

$$\langle v, \partial_\alpha v \rangle = 0 ; \quad \langle \partial_\alpha v, \partial_\beta v \rangle = 0 . \quad (22)$$

If the metric is positive definite, one can show [10] that $(\bar{X}^I, \partial_\alpha X^I)$ is an invertible $(n+1) \times (n+1)$ matrix, and one defines

$$\bar{\mathcal{N}} = [\bar{X}^I, \partial_\alpha X^I] [(\bar{X}^I, \partial_\alpha X^I)]^{-1} , \quad (23)$$

from which one proves $\mathcal{N} = \mathcal{N}^T$ and $\text{Im } \mathcal{N} < 0$. If a prepotential exists then this leads to Eq. 19.

It turns out that the following 4 conditions are equivalent: 1) $(X^I, \partial_\alpha X^I)$ invertible; 2) special coordinates are possible; 3) $e_\alpha^A = \partial_\alpha \left(\frac{X^A}{X^0} \right)$ (for a choice of the coordinate "0", and $A = 1, \dots, n$); 4) a prepotential $F(X)$ exists.

For the last statement, one first notices that (X^I/X^0) is independent of \bar{z} , such that from the functions $F_I(z, \bar{z})$, one can define holomorphic functions

$$F_I(X) \equiv X^0 \frac{F_I}{X^0} \left(z \left(\frac{X^A}{X^0} \right) \right) . \quad (24)$$

The constraints Eq. 22 then imply

$$\begin{pmatrix} X^I \\ \partial_\alpha X^I \end{pmatrix} \partial_{[I} F_{J]} (X^J, \partial_\alpha X^J) = 0 , \quad (25)$$

from which it follows that in any patch $F_J = \frac{\partial}{\partial X^J} F(X)$ for some $F(X)$.

Finally, we have proven in [10] that for any special Kähler manifold there exists a symplectic transformation to a symplectic basis such that the conditions 1) are satisfied.

This implies that all the Kähler manifolds which are special, can be constructed from prepotentials. Indeed for symplectic vectors which satisfy the necessary conditions, we in general still need a symplectic transformation to obtain a formulation with a prepotential, but the Kähler potential, defined in Eq. 11, is obviously invariant under such a transformation.

On the other hand, such a symplectic transformation is not necessary an invariance of the action, so the models without prepotential are necessary for a complete description of the matter couplings of $N = 2$ vector multiplets, as it was shown in [8].

Finally, let me remark that the symplectic transformations as used above act on the vector define new vectors \tilde{Z}^I . Therefore one can often associate to these also a coordinate transformation if the coordinates z^α are defined in terms of Z^I (e.g. the special coordinates). In the latter case, the symplectic transformations induce coordinate transformations. As in the early papers of special geometry one always used special coordinates, the symplectic transformations were usually considered in combination with these coordinate transformations. If that is done, then the Kähler potential is not necessary invariant. The combined transformations which leave the Kähler potential invariant then define 'duality symmetries' [3, 17]. But one should clearly distinguish these transformations. In [18] this has been discussed in a more general context.

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